Frequency-domain Modeling Using Implicit and Explicit Finite-difference Operators

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SUMMARY

Several techniques have been developed to modeling the wave equation. In this paper we introduce the frequency-domain finite-difference (FDFD) modeling and its implementation using the explicit (2nd and 4th) and implicit (nine-point) centered difference operators for solving the scalar Helmholtz equation. We discuss some proprieties of the resulting linear system after the discretization of the wave equation and show that our implemented nine-point implicit scheme produces a much more accurate result in comparison with the 2nd and 4th schemes for seismic wave simulations in the frequency domain.
Introduction

The hydrocarbons industry makes use of computational intensive algorithms such as migration and full waveform inversion to provide an image of the subsurface. The results of these are directly dependent on the accuracy of the method used in the seismic imaging. Algorithms for the seismic imaging can be divided into two main groups: those formulated in the time domain and those that perform in the frequency domain, where numerical discretization of the Helmholtz equation results in an implicit discrete system that requires the use of linear solvers. The frequency-domain finite-difference (FDFD) modeling offers several advantages over traditional time domain methods, being the parallelization on the spectrum of frequencies the main advantage. In this paper we present the implementation of the FDFD method using a parallelization on the band frequencies to solve the Helmholtz equation. The sparse LU decomposition is used to find the solution of the linear system at each frequency. In the numerical example section, we present the results for a three-layer model and demonstrate the superior accuracy of the nine-point grid in comparison with the 2nd and 4th order schemes.

Problem Formulation

We begin with the scalar wave equation in 2D. By applying a Fourier transform in time, one obtains the following Helmholtz equation

\[
\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} \right] p = -s, \tag{1}
\]

where \( p(x, z, \omega) \) is the wave pressure field, \( \omega \) is the temporal angular frequency, \( c = c(x, z) \) is the P-wave velocity and \( s = s(x, z, \omega) \) is the source term.

Discretization and implementation

We discretize the Eq. (1) using an explicit finite difference scheme based on 2nd and 4th order accurate centered difference operators. The nine-point implicit finite-difference scheme (Figure 1a), proposed by Chu and Stoffa (2012), is additionally implemented to improve the accuracy of the finite difference simulations without dramatically increasing the memory cost after the LU decomposition. After discretization of Eq. (1) on \( \Omega \) and applying the sponge boundary conditions proposed by Shin (1995) we obtain the following linear system of equations:

\[
Ap = -s, \quad A \in \mathbb{C}^{N \times N}, \quad p, s \in \mathbb{C}^N, \tag{2}
\]

where \( N = N_x \times N_z \) and \( A \) is the impedance matrix with dimension \( N \times N \). In each line of \( A \) the 2D locations from the FD stencils are mapped to the 1D index \( k \) using \( k = (i-1)N_z + j \).

![Figure 1](image)

*Figure 1* (a) Finite-difference stencils for the 2D Laplacian operator and (b) structure of the matrix impedance of the conventional second-order (green), fourth-order (red) and nine-point (blue) scheme.

In general, the structure of \( A \) is sparse and square (Figure 1b). Complex values are confined to the central diagonal and it is an indefinite matrix. Considering different computational advantages, a variety of methods direct and iterative have been proposed to solve the system give by Eq. (2). In this work,
the MUMPS solver, which is based on a multifrontal method and direct sparse lower-upper factorization (Amestoy et al., 2001) is used to determine the linear system in Eq. (2).

**Numerical example**

We use a three-layer model to compare the nine-point implicit scheme with the 2nd and 4th order explicit schemes. The depth of the layers are 1.2, 2.0 and 3.0 km with velocities of 2400, 3200 and 4000 m/s, respectively. The Ricker wavelet is used as the source function and is located 200 m above of the first layer boundary, the receivers are 100 m below of the surface. We computed frequency responses up to 30 Hz and then transformed them into the time domain. Figure 2 shows the shot records computed with the three different schemes cited in the implementation section of this work, the nine-point stencil produces a much more accurate result (Figure 2c) reducing significantly the dispersion generated in the application of the others schemes due to the insufficient sampling rate in space ($\Delta x = \Delta z = 20m$).

The result for the fourth-order scheme was obtained with a higher cost in terms of computing time (1.5 times) and memory consumption (168%) with respect to a 2nd order grid. In comparison, the computational time of the nine-point scheme was approximately 0.5 times that of the 2nd order scheme and the memory cost after the LU decomposition increased by 50% for each frequency.

![Figure 2](image)

**Figure 2** Shot records for a three-layer model computed in the frequency domain by (a) the conventional second-order, (b) fourth-order, and (c) the nine-point implicit finite-difference scheme.

**Conclusions**

We show the superior accuracy of the nine-point grid compared with the conventional explicit finite-difference scheme. The result with nine-point implicit method is better than those obtained by the 4th order grid and the computational cost is similar to the 2nd order scheme. We can also conclude that modeling in the frequency domain allows the implementation of parallel programming, which is reflected in a high computational efficiency.

**Acknowledgements**

D. E. Revelo acknowledges the Brazilian agency CAPES for a M.Sc. scholarship. R. Pestana would like to thank CPGG/UFBA, the CNPq and INCT-GP/CNPq for supporting the development of this work.

**References**

