The estimation of the physical properties of subsurface objects using Cole-Cole equation

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Ground Penetrating Rader (GPR) is a nondestructive testing method to visualize shallow subsurface using reflection of downgoing electromagnetic (EM) waves generated on the surface. The processing of GPR data has been developed using that of reflection seismology and the method is applied to survey wide area in an expeditious way, GPR has been widely used in many engineering fields. Quantitative analyses based on physical properties of buried targets have, however, not been well attempted in spite of increasing interest to the identification of subsurface materials in recent years. Since the combination of conductivity, permittivity, and magnetic permeability contrasts of material discontinuities causes the attenuation and the dispersion of electromagnetic waves that propagate through, reflected electromagnetic signals could be utilized to the material identification. In this study, we employed the Cole-Cole equation to analyze the attenuation and the dispersion of electromagnetic reflection signals for possible material identification using what are called the Cole parameters, i.e., physical properties. Since there are many materials for which the Cole parameters are not estimated, we employed a method using reflection signals for a two-layered earth model, that has a top layer of a material with the known Cole parameters and the second layer for which the Cole parameters are estimated. We use the dry soil as the second layer to estimate the physical properties using the Cole-Cole equation. Although we use very wide range of frequency in this study as well to discuss the optimum frequency band, we get good estimation result from the frequency band from $10^7 H_z$ to $10^8 H_z$, which is usually used in GPR.

1. INTRODUCTION

GPR survey is a nondestructive testing method to visualize shallow subsurface using reflection of downgoing electromagnetic (EM) waves[1]. Travel-time of reflection signals tells us the position and the depth of objects in the subsurface. Since it is possible to survey wide area in an expeditious way, GPR has been widely used in many engineering fields. On the other hand, it has been thought to be difficult for the conventional GPR survey to estimate physical properties in the subsurface. However, the information about specific properties of the subsurface targets could help interpretation in GPR survey. The possibility to estimate the physical properties using EM waves reflection was presented[2] previously using the Cole theory[3][4], but the model and the frequency band used in their study were not realistic for GPR utilization. GPR surveys based on the Cole theory need to be taken into account.

The previous studies have focused on chemico-physical application, the Cole theory has mainly been applied for laboratory-scale experiments. In GPR survey, the frequency of emitted EM waves depends on the shape and the size of antenna, and we need to pay attention to the optimal frequency range of analysis in the application of the Cole theory. The frequency band from 10^{10}Hz to 10^{12}Hz was presented in the previous study[5], but may not be the optimum frequency for GPR survey, since the target material may not be in the laboratory scale. The optimum frequency range as well as the Cole parameters of realistic earth materials should be visited.

We started considering a new way of GPR surveys using the Cole theory to explore near surface in a quantitative manner. Also, our simulation model is changed to a realistic one, i.e. from a methanol and ethanol model to the one using a dry soil layer, for a frequency band extended towards low frequencies. Since there may be no Cole parameters for realistic earth materials, we started estimating the Cole parameters to the dry soil interfacing to the first layer of methanol, for which the Cole parameters are known, using electromagnetic reflections.

2. THEORY

In this study, we notice dielectric relaxation, which arises in the microwave range and focus on the Cole theory to estimate physical properties in the subsurface. In this theory, we can calculate specific parameters depend on target media using complex relative permittivity of the medium. According to the Cole theory, dielectric relaxation is represented by the empirical formula.

$$\varepsilon_r^* = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 + (i\omega\tau)^{1-\alpha}} \quad (1)$$

In this equation, ε_r is the complex permittivity, ε_0 and ε_∞ are the static and infinite frequency dielectric constants, τ is the relaxation time and α is the parameter, which indicates the distribution of the relaxation time and ranges from 0 to 1. Since these values are inherent, these values enable us to estimate the physical properties of subsurface targets. Using the equations shown below (equations 2-6), we calculate the complex relative permittivity from amplitude reflectance and the phase change obtained by reflection of EM waves. Then we plot the complex relative permittivity on the complex plane and estimate both ε_0 and ε_∞ by two intersection points on the real axis. τ is estimated by the frequency at which the imaginary part of complex relative permittivity showed its peak.

$$n_2 = \frac{(1-R)n_1 + 2\sqrt{R}\sin\varphi \cdot \kappa_1}{1 + 2\sqrt{R}\cos\varphi + R} \quad (2)$$

$$\kappa_2 = \frac{-2\sqrt{R}\sin\varphi \cdot n_1 + (1-R)\,\kappa_1}{1 + 2\sqrt{R}\cos\varphi + R} \quad (3)$$

$$\varepsilon_r^* = \varepsilon_R^* + i\varepsilon_I^* \quad (4)$$

$$\varepsilon_R^* = n^2 - \kappa^2 \quad (5)$$

$$\varepsilon_I^* = 2n\kappa \quad (6)$$

3. SIMULATION MODEL

Figure 1 shows our numerical model. We use the Ricker wavelet as a source, and change the peak frequency involved in the Ricker wavelet as shown in Table 1. After applying the fast Fourier-transform (FFT), we use the amplitude data only half-value width of the peak, so in previous study, there were wide range frequency bands not used in the estimation. In this study, we set the peak frequency of Ricker wavelet as the treated frequency range become consecutive after FFT like Figure 2.

4. RESULTS

We calculate the complex relative permittivity of dry soil which is deeper layer, and plot it on the complex plane as shown in Figure 3. Figure 4 indicates both of real part and imaginary part of the complex relative permittivity plotted against frequency. Figure 5 indicates the Cole plot using only for the limited frequency bands, which is used in the conventional GPR survey. Then we find the circle which corresponds with the plot in Figure 5 to estimate the dry soil's Cole parameters. The approximate circle is added in Figure 5.

5. DISCUSSIONS

The estimated values for the dry soil are shown in Table 2. In THEORY section, we showed α is the parameter, which indicates the distribution of the relaxation time and ranges from 0 to 1, but in our estimation, the value of α is over 1, and ε_0 is also bigger than the value of methanol and ethanol. However, the value of the relaxation time τ and the relative permittivity at the very high frequency ε_∞ matches with the result of the case in which we estimate the value of methanol, thus this estimation seems to be reasonable. And it is showed that the frequency band, which is usually used in GPR survey, is effective for the target in the subsurface.

On the other hand, the accuracy of the estimated values is unclear because there are few experiments for the object under the ground using Cole plot. Thus, it is needed to do experiments for the rock and soil etc. for further application of the Cole plot.

In Figure.4, the imaginary part against frequency show similar behavior as dielectric relaxation by drawing the envelope at the each peak. Since the amplitudes are different at each frequency and there are narrow ranges, which are not included in this simulation (Figure.2), the behavior of the imaginary part is unstable. Thus, it is needed to use the wave whose amplitude spectral is flat between the frequencies used in estimation.

6. CONCLUSION

In this study, we try to estimate the dry soil's specific physical properties using the Cole plot. Although the two estimated values ε_0 and α are different from the case in which we estimate the values of methanol, the relaxation time τ and the relative permittivity at very high frequency ε_∞ are almost the same. Thus, this estimation is seems to be reasonable.

On the other hand, the accuracy of the estimated values is not unclear because there are few true values, which are measured by the experiment.

It is needed the experiment in which the physical properties of the object under subsurface are measured.

Moreover, the use of the wave which has the flat peak at the range of frequency used in the estimation is also needed to stabilize the behavior of the imaginary part of the complex relative permittivity against frequency.

REFERENCES

1) Kikuchi, K., Mikada, H., Takekawa, J. (2017): Improved Imaging Capability of Phased Array Antenna in Ground Penetrating Radar Survey, 79th EAGE Conference and Exhibition 2017, doi:10.3997/2214-4609.201701121

2) Suda, M. (2017): A fundamental research on complex permittivity of conductive material using reflected electromagnetic waves, Kyoto University Graduation Thesis. 27 pages

3) Kenneth S. ColeRobert H. Cole (1941) : Dispersion and Absorption in Dielectrics Ⅰ. Alternating Current Characteristics. JOURNAL OF CHEMICAL PHYSICS (vol9,pp341-351)

4) Kenneth S. ColeRobert H. Cole (1942) : Dispersion and Absorption in Dielectrics Ⅱ. Direct Current Characteristics. JOURNAL OF CHEMICAL PHYSICS (vol10,pp98-105)

5) Kimura, K. (2018): Fundamental research on the estimation of complex relative permittivity using reflection of electromagnetic waves, Kyoto University Graduation Thesis. 30 pages

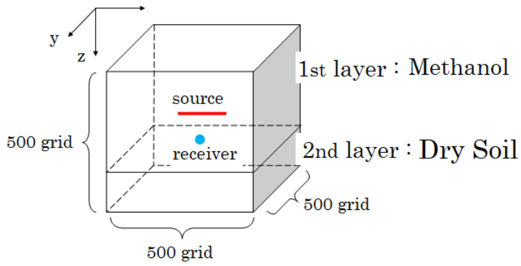

Figure 1 : Simulation model

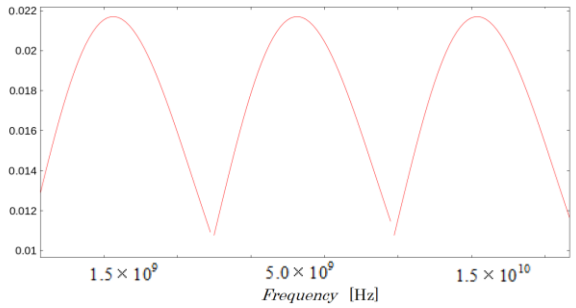

Figure 2 : The wave in frequency domain

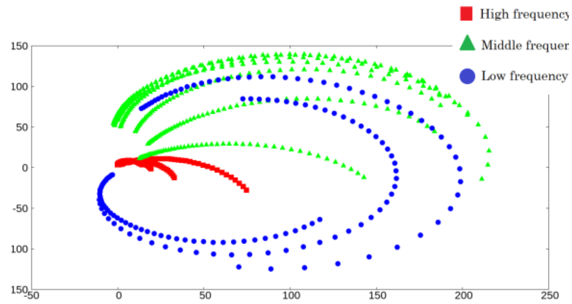

Figure 3 : Cole plot

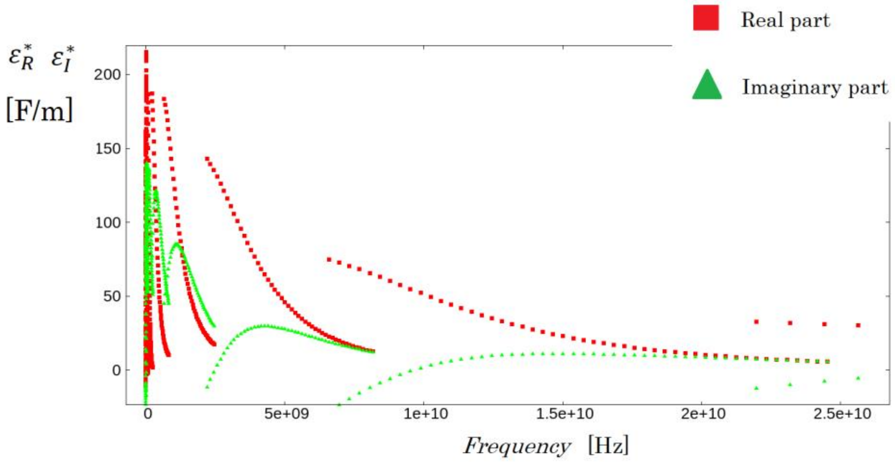

Figure 4 : Real and imaginary part of complex relative permittivity against frequency

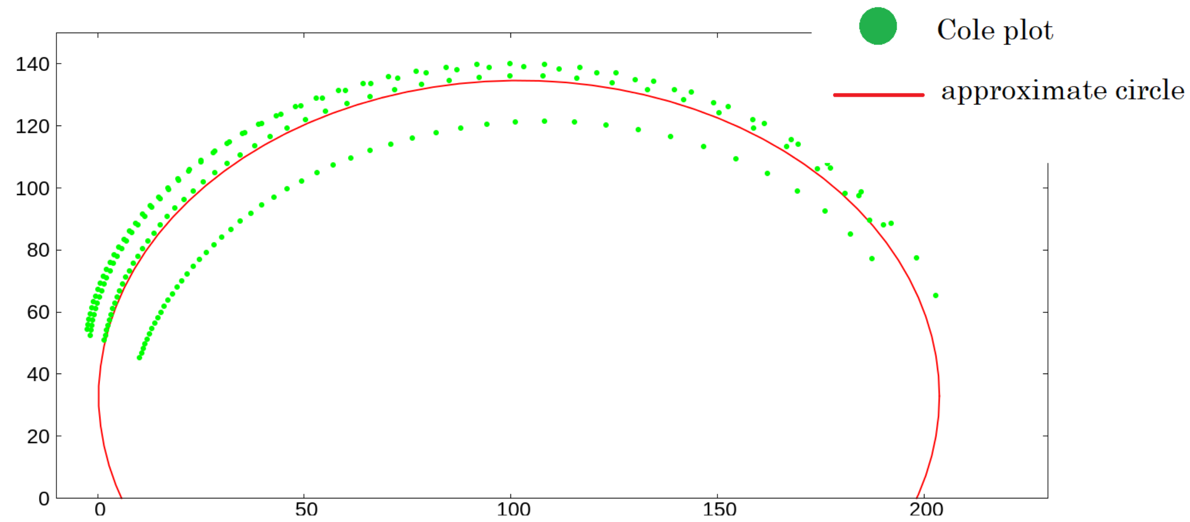

Figure 5 : Cole plot using only frequency band which is usually used in GPR survey and the approximate circle

Table 1 : peak frequency, grid distance and time interval in each case.

model	Peak frequency [Hz]	Grid distance [m]	Time interval [s]
1	1.5×10^{12}	5.0×10^{-7}	$5/3\times10^{-15}$
2	5.0×10^{11}	1.5×10^{-6}	5.0×10^{-15}
3	1.5×10^{11}	5.0×10^{-6}	$5/3\times10^{-14}$
4	5.0×10^{10}	1.5×10^{-5}	5.0×10^{-14}
5	1.5×10^{10}	5.0×10^{-5}	$5/3\times10^{-13}$
6	5.0×10^{9}	1.5×10^{-4}	5.0×10^{-13}
7	1.5×10^{9}	5.0×10^{-4}	$5/3\times10^{-12}$
8	5.0×10^{8}	1.5×10^{-3}	5.0×10^{-12}
9	1.5×10^{8}	5.0×10^{-3}	$5/3\times10^{-11}$
10	5.0×10^{7}	1.5×10^{-2}	5.0×10^{-11}
11	1.5×10^{7}	5.0×10^{-2}	$5/3\times10^{-10}$
12	5.0×10^{6}	1.5×10^{-1}	5.0×10^{-10}
13	1.5×10^{6}	5.0×10^{-1}	$5/3\times10^{-9}$
14	5.0×10^{5}	1.5	5.0×10^{-9}
15	1.5×10^{5}	5.0	$5/3\times10^{-8}$

Table 2 : the estimated values of dry soil and methanol

	ε_0	ε_∞	τ	α
Dry soil	198.25	5.73	2.89×10^{-9}	1.79
Methanol	18.24	1.28	7.67×10^{-11}	0.029